

```

> Rect:=proc(f,N,a,b)
> local nbrect,i,S,x,n;
> nbrect:=[seq(N*i,i=1..N)]:
> S:=seq(student[leftbox](f(x),x=a..b,n),n=nbrect):
> plots[display](S,insequence=true);
> end:
> Rect(x->x^2,5,0,1);

```

```

> Rect(x->cos(x),10,0,Pi);

```

```

> airerect:=proc(f,N,a,b)
> local k;
> Sum((b-a)/N*f(a+k*(b-a)/N),k=1..N)=evalf(sum((b-a)/N*f(a+k*(b-a)/N),k=1..N));
> end:
> airerect(x->x^2,10,0,1);

```

$$\sum_{k=1}^{10} \left(\frac{1}{1000} k^2 \right) = .3850000000$$

```

> airerect(x->cos(x),10,0,Pi);

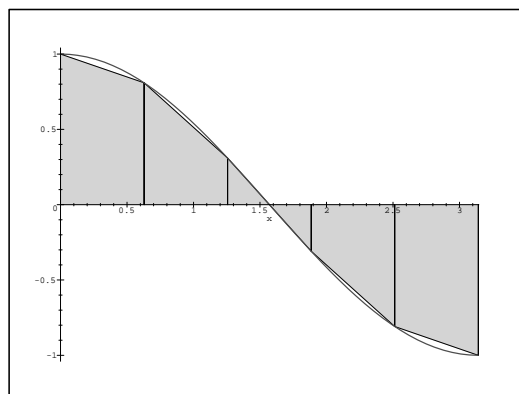
```

$$\sum_{k=1}^{10} \left(\frac{1}{10} \pi \cos\left(\frac{1}{10} k \pi\right) \right) = -.3141592654$$

```

> Trap:=proc(f,N,a,b)
> local TV,TR,k,av,ap,T,C;
> TV:=NULL: TR:=NULL:
> for k from 0 to N do
> av:=a+k*(b-a)/N:
> ap:=a+(k+1)*(b-a)/N:
> T:=[[av,0],[av,f(av)],[ap,f(ap)],[ap,0]]:
> TV:=TV,plot(T,filled=true,color=wheat):
> TR:=TR,plot(T,color=black,thickness=2):
> od:
> C:=plot(f(x),x=a..b,thickness=3):
> plots[display](TV,TR,C);
> end:
> Trap(x->cos(x),5,0,Pi);

```



```

> Trapanim:=proc(f,N,a,b)
> local nbtrap,i,S,n;
> nbtrap:=[seq(N*i,i=1..N)]:
> S:=seq(Trap(f,n,a,b),n=nbtrap):
> plots[display](S,insequence=true);
> end:
> Trapanim(x->cos(x),3,0,2*Pi);

```

```

> Riem:=proc(f,a,b)
> local k;
> Limit(Sum(((b-a)/n)*f(a+k*(b-a)/n),k=1..n),n=infinity)=limit(sum(((b-a)
> )/n)*f(a+k*(b-a)/n),k=1..n),n=infinity);
> end:

```

> Riem(x->x^2, -5, 2);

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \left(7 \frac{(-5 + 7 \frac{k}{n})^2}{n} \right) = \frac{133}{3}$$

> int(x^2, x=-5..2);

$$\frac{133}{3}$$

```
> airetrap:=proc(f,N,a,b)
> local k;
> (b-a)/N*((f(a)+f(b))/2+Sum(f(a+k*(b-a)/N),k=1..N-1))=evalf((b-a)/N*((f
> (a)+f(b))/2+Sum(f(a+k*(b-a)/N),k=1..N-1)));
> end:
> evalf(Pi);
```

3.141592654

> 4*airerect(x->sqrt(1-x^2), 10000, 0, 1);

$$4 \left(\sum_{k=1}^{10000} \left(\frac{1}{100000000} \sqrt{100000000 - k^2} \right) \right) = 3.141391481$$

> 4*airetrap(x->sqrt(1-x^2), 10000, 0, 1);

$$\frac{1}{5000} + \frac{1}{2500} \left(\sum_{k=1}^{9999} \left(\frac{1}{10000} \sqrt{100000000 - k^2} \right) \right) = 3.141591478$$

```
> mini:=proc(u,p)
> local k;k:=0;
> while (evalf((u^(2*k+3))/(2*k+3)))>(10^(-p)) do
> k:=k+1;
> od:
> end:
> greg:=proc(n,a,p)
> local S,k; S:=0;
> for k from 0 to n+1/2 do
> S:=S+evalf(((1)^k*a^(2*k+1))/(2*k+1),p+1);
> od;
> S;
> end:
> mini(1/5,100);
```

69

```
> Digits:=100;
> 4*(4*greg(69,1/5,100)-greg(69,1/239,100));
```

3.1415926535897932384626433832795028841971693993751058209749445923\
07816406286208998628034825342117064

```
> 4*(4*greg(69,1/5,100)-greg(69,1/239,100))-eva
> lf(Pi,100);
```

-.4 10⁻⁹⁸